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ABSTRACT

This review considers the question as to whether there are generic problem-solving skills that cut across fields or whether the skills are so embedded within specific fields that they can be identified only within the contexts of those fields. To answer this question, an attempt was made to define both "problems" and their "solutions." Then the evidence for the existence of general problem-solving skills that are independent from any specific field was examined. The analyses of skills within disciplines were reviewed to see if the skills are common across fields. Finally, the implications of the research for the assessment of problem-solving skills were studied. In general, it was concluded that similar skills are used in different fields, but that their implementation is so dependent on mastery of the specific fields that any assessment of problem-solving skills would best be conducted within the fields. (Author)

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RESEARCH REPORT

REVIEW OF PROBLEM-SOLVING SKILLS

Leonard L. Baird

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Review of Problem Solving Skills

Leonard L. Baird

November, 1982

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Abstract

This review considers the question as to whether there are generic problem-solving skills that cut across fields or whether the skills are so embedded within specific fields that they can be identified only within the contexts of those fields. To answer this question, an attempt was made to define both "problems" and their "solution." Then the evidence for the existence of general problem solving skills that are independent from any specific field was examined. Then the analyses of skills within disciplines were reviewed to see if the skills are common across fields. Finally, the implications of the research for the assessment of problem-solving skills were discussed. In general, it was concluded that similar skills are used in different fields but that their implementation is so dependent on mastery of the specific fields that any assessment of problem-solving skills would best be conducted within the fields.

Review of Problem Solving Skills

Each of us solves problems every day, although we usually don't think of our activities in those terms. For example, we solve problems when we plan the best route to work, or make up a grocery list. More complex examples might include a mechanic diagnosing a strange noise in an engine, and a sales manager thinking of ways to boost the sales of a slagging product. The highest level of problem solving is found in the work of the relatively small group of scientists, engineers, writers, artists, and thinkers who are at the forefront of work in their fields.

As these examples may also suggest, virtually all of us would like to improve our problem-solving skills, to become more effective and efficient in dealing with the simple and difficult problems we face each day. If it were possible to identify particular strategies or skills that were effective in solving problems, it might be possible to learn them. From a different perspective, if it were possible to identify these skills, and to assess the relative levels of skill of individuals it would be possible to identify especially capable problem solvers. These individuals could then be selected for various positions for which their skills would be appropriate.

More formally, the central question of this review is, "Are there generic problem-solving skills that cut across fields, and which could be assessed, and/or taught?" Or are the skills so embedded within the context of specific fields that they can be assessed only within those fields?"

To answer this central question, we need to deal with several issues. First, we shall attempt to define the domain, i.e., determine what constitutes the realm of "problems" and their "solution." Some have argued that if a procedure exists for solving a class of problems, that the use of these

procedures is merely mechanical, and does not represent real problem solving. Secondly, we will examine the evidence for the existence of general problem solving skills that are not specific to any particular domain. Then we will review the analyses of how problems are solved within various disciplines to see if the same skills are common across fields. Finally, the implications of the research findings for the assessment of problem solving skills will be discussed.

The literature bearing on problem solving is so voluminous that it is impossible to review even a major part of it. Consequently, the literature reviewed in the following pages represents a selection. However, the emphasis was placed on ideas that have baen subject to epirical test or which in some way have research support.

What is a Problem?

Mayer (1977) has reviewed research on thinking and problem solving and concludes that a problem has givens, goals, and obstacles. The given state is the current fact, situation, or condition. The goal state is a different, more valued fact, situation or condition. The obstacles are the difficulties that must be surmounted before the given state can be changed. The operations that will convert the current state to the goal state are the solution to the problem. This conception is quite abstract, but is very serviceable for our needs.

One of the important characteristics of givens, goals, and solutions is their clarity (Reitman, 1965). That is, a given state may be well-defined or very ill-defined, as can the end state and the solution. For example, in the problem of adding a column of numbers the givens, goals, and solutions are all well defined. (In fact, because certain problems such as arithmetic problems

have known algorithms for their solutions, some observers have argued they are not true problems at all. However, real-life arithmetic problems, such as balancing a checkbook or calculating one's income tax, often have many of the complexities and difficulties of other problems. The definition used here will be applied as broadly as possible, while recognizing that problems can obviously vary in their importance and difficulty.) Simon (1973) has made a similar distinction between "ill-structured" and "well structured problems.

Another consideration is the specificity of the goal state and the solution. That is, there may be only one solution to a problem or there may be many possible solutions. This can be relatively unrelated to the difficulty of the problem. For many scientific problems, for example, there may be only one solution, but the steps to solution may be very ill-defined. On the other hand, a relatively simple problem, such as what to serve for dinner may have many possible solutions. Thus there are continua of clarity of problem conditions and steps to solution, as shown in Figure 1.

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Insert Figure 1 about here
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Problem finding. Related to the issue of the clarity of the definition of a problem is the recognition that a problem exists. That is, some people have skills in discovering problems. Getzels and Csikszentmihalyi (1975) call this "problem finding." They note that "At one extreme there are presented problem situations where the problem has a known formulation, a routine method of solution, and a recognized solution; here a person needs only to follow established steps to meet the requirements of the situation. At the other extreme there are discovered problem situations where the problem does not yet have a known

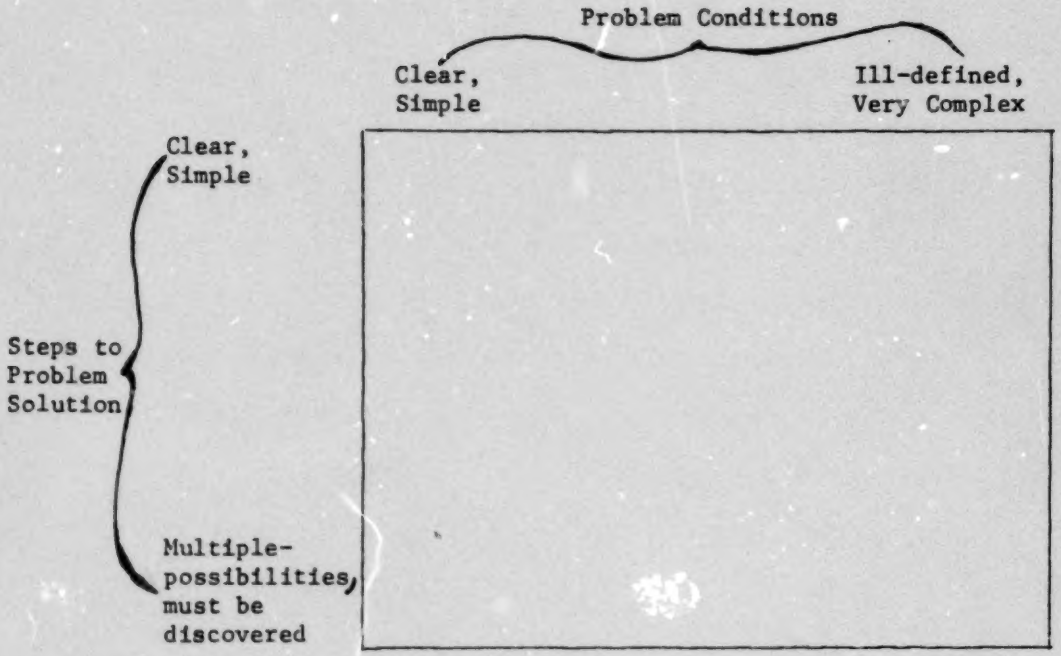

Figure 1. Continua of Clarity of Problem Conditions and Steps to Solution

formulation, a routine method of solution, and a recognized solution; here the person must identify the problem itself, and there are no established steps for satisfying the requirements of the situation" (1975, p. 101-102). Examples of discovered problem situations include the work of painters, sculptors, and writers who are faced with the proverbial empty canvas or blank page, and scientists examining apparently random data. Mackworth (1969) has emphasized the importance of the distinction between problem solving and problem finding.

> ...there is an all-important qualitative difference between problem solving and finding. It is clear that problem solving is a choice between existing programs or sets of mental rules--can sometimes even come from discarded programs that proved unsuitable for the initial question. The creative scientist often appears to stumble across new problems. To do this he must start by noting the need for looking at data in a fresh way for a given purpose. The alert problem finder must then be able to produce these changes in his mental coding arrangements. In a sense, he can only do this by devising new mental programs or plans and realizing they are more suitable for relating the facts than the existing mental rules. Quite expectedly these new programs may also apply to other problems which can now be tackled for the first time.

Mackworth developed a table, reproduced here as Table 1, to highlight the comparisons between problem solving and problem finding.

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Insert Table 1 about here
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In summary, problems can vary in clarity, both in the given situation, and in the steps needed for their solution. Perhaps the most important type of problems are those that are implicit in situations, and must be "discovered," "found," or "defined."

Greeno's typology. Greeno (1978) has proposed a typology of problems based on a review of the experimental tasks used in problem solving studies. Greeno calls the first type "problems of inducing structure." Such problems include

Table 1

Problem Solving and Problem Finding by Humans

	Problem Solving	Problem Finding
Definition	Problem solving is the selection and use of an existing program from an existing set of programs.	Problem finding is the detection of the need for a new program by comparing existing and expected future programs.
Objective	To choose correctly between existing programs--in order to select the one program that effectively elicits the required actions from a set of possible responses.	To choose correctly between existing and expected future programs--in order to devise new programs and to realize that one or more of these would be more suitable than any of the existing programs in eliciting the required actions.
Method	Experiment more than thought minimizes the mismatch between the desired and apparent actual states.	Thought more than experiment minimizes the mismatch between the desired and apparent actual states.
Outcome	Success is the discovery of one specific acceptable answer to one well-defined problem.	Success is the discovery of many general questions from many ill-defined problems.

verbal analogy problems which require the subject to organize the presented words into a general structure or pattern of relationships. "Series-extrapolation problems" also require inducing structure (e.g., "What comes next in this series? 1, 1, 2, 4, 3, 9, 4 ?" in which a number is followed by its square).

Greeno's second type of problem is "transformation"--problems in which the task is to take a given situation, operate on it or change it in some way to another state which is the goal. Such problems include "move" problems such as the Tower of Hanoi and the Luchin's Water Jug problem. In other problems, the objects in the problems can only be changed according to a specific set of rules. In his review, Greeno emphasized the importance of analyzing situations, which "...involves a process of identifying features of the situation that are relevant to later outcomes." Obviously, thorough familiarity with the operations needed to solve the problem are needed. The ability to formulate a plan that will guide the selection of operators is also important.

Greeno's third type of problem is "problems of structure and transformation." These problems involve understanding the instructions to problems, or correct representation of the problem situation. Related to the importance of representation is general knowledge that allows the problem solver to put the problem into a general context. Then, of course, relating the understanding of the general concepts to the goal of the problem is a critical variable. Greeno also notes the importance of fluency in generating possible solutions, memory of related problems, the ability to work within the constraints of the problem, and having the needed algorithms that facilitate solving the problem.

Although Greeno's analysis is helpful in organizing the evidence about some of the experimental tasks used in studies of problem solving, it is

limited in several ways. First, most of the experimental tasks are designed to yield clear, interpretable data, usually involving discrete steps toward a solution. Although this provides the experimenter with a highly controlled, clear situation that provides data, the tasks are, consequently, highly artificial. It is unclear how, or even whether, these tasks bear resemblance to real-life problems. Second, the descriptions of the underlying characteristics of the problems are so abstract that it is hard to apply them. For example, "understanding the problem" is obviously important, but what does it mean in any specific case? Third, the typology itself is based on artificial experimental tasks. It is unclear whether this typology would resemble a typology based on real-life problems.

In summary, there are many ways to define "problems." They can be seen in terms of the clarity of their requirements, the clarity of the steps needed to reach a solution, and the extent to which they are well-known problems in a domain of activity or need to be "found" in the problem situation. They vary in terms of the kinds of basic operations that are needed to solve them, and, especially relevant to the pages that follow, the extent to which they have verisimilitude and generalizability. As we examine the research on problem solving in the following pages, it will be useful to keep these points in mind.

What is "solving" a problem?

Students of problem solving often make a distinction between algorithms and heuristics. This distinction, first elaborated by Polya (1952) is based on the observation that some techniques, if followed carefully, guarantee a solution to a problem, while others, although frequently leading to a correct solution, do not always do so. The former, called algorithms, are often

detailed step-by-step procedures. The later, called heuristics, are typically general strategies that can be applied in a wide variety of situations when the solutions are uncertain. An algorithm is based on specific knowledge, such as knowing the quadratic equation or the Pythagorean theorem.

Heuristics can be quite general strategies, such as making a plan for proceeding or checking the accuracy of the basic assumptions, and are usually not based on specific knowledge. As noted previously, some early writers made much of this distinction, regarding the use of algorithms as a mechanical application of rules that does not really deserve to be called problem solving. True problem solving was taken to mean grappling with unusual situations with no obvious way to proceed except through general heuristics. More recent research has eroded this distinction (Greeno, 1980). Rather, most current writers on problem solving agree that there is a continuum from very specific, knowledge based procedures, such as following a recipe when one is faced with the problem of baking a cake, to very general procedures or approaches, such as attempting to think of a metaphor for a scientific problem. As Greeno (1980) points out:

> The point of this part of my discussion is to illustrate that the major cognitive components of problem solving turn out to be present in many situations in which we are not accustomed to honoring successful performance as instances of problem solving. It is certainly important to distinguish between: (1) situations in which the performer has relatively specific knowledge that makes problem solving quite easy; and (2) other situations in which the performer must resort to more general knowledge and procedures to solve a problem. However, the specificity of available knowledge is a matter of degree, not kind. It is seriously misleading to label performance in some situations as problem solving and in other situations in which the same kinds of cognitive processes occur as not involving problem solving. A continuum should be called a continuum, not a dichotomy. (p. 12)

This is a fundamental point which goes beyond the experimental study of problem solving per se. That is, to what extent does "problem solving" appear

when procedures exist within a profession or area of skill that can be used to attack the problem? For example, when a new patient enters a doctor's office, there are standard procedures for narrowing in on a diagnosis and a subsequent prescription and prognosis. The physician will usually obtain a medical history, a record of current symptoms, and will then decide on a series of tests--temperature, pulse rate, blood pressure, throat culture, etc.--all according to standardized procedures dictated by the medical profession. Similar standardized procedures exist within most professions and scientific and scholarly disciplines. Standard methods are also part of skilled trades. The electrician checks out the circuitry in a house, the mechanic looks for signs of a malfunction, the loan officer estimates the degree of risk in evaluating a loan application. All of these procedures follow formal or informal rules learned as part of the training for a profession or trade. The point is that people trained in various areas are taught fairly standardized procedures for solving the typical problems they will encounter in their work. To the extent the profession or area of skill has procedures that will lead to the correct solution of the problems its practitioners face, the less will "heuristics" come into play and the more will knowledge-based "algorithms" come into play. Obviously these procedures will generally be specific to each profession and situation, with little, if any, generality across professions or situations. For example, an automobile mechanic's compression test to see if an engine's cylinder is leaking is quite distinct from checking a backup light switch. And, as many of us have found, procedures may be even more specific. For example, the mechanical procedures designed for a large Pontiac may not be the same as those for a small Toyota. Obviously the efficient mechanic, the

incisive lawyer, and the skilled internist are good "problem solvers." When we have an automotive, legal, or medical problem, we seek these people out because they can solve our problems. One of the reasons we do not turn to amateurs for such problems is that they do not have the knowledge and procedures to lead to a timely, efficient, and effective solution to our problems. Amateurs would have to rely on very general heuristics, would require a long time to come to a solution, and would be unsure that their solution was the correct one. However, if they did find the correct solution, would this represent a greater degree of "problem solving" skill than that shown by the expert in the field?

The point is that most problems are solved by discipline-specific procedures and techniques. This returns us to the central question of whether there are generic problem solving skills that cut across disciplines. Or, if there are similarities across disciplines would the general rule be so broad as to be useless without discipline-specific knowledge and detailed procedures? For example, one rule might be to "formulate competing hypotheses and seek evidence that would rule out one or all of them." However, to test out the hypothesis that a fuse in a fuse block or a solenoid elsewhere is causing a short circuit, a mechanic needs to know a good deal about the electrical system of the particular make of car involved, and needs to know how to use a volt-ohm-milliammeter. A physician attempting to determine whether a blurring of vision is due to glaucoma or a cataract needs to know the appropriate medical procedures to follow.

As Larkin (1980) points out:

> The preceding comments suggest that there may indeed be some general strategies (i.e., some major features) that are seen in skillful problem solving in a variety of disciplines. However, these strategies cannot be implemented without a considerable amount of domain-specific knowledge. For example, in order to use means-ends analysis, the solver studied by Simon

> and Simon had to know how to assess the difference between the equations she had and the equations needed to solve the problem. She had to know what kind of algebraic manipulations would reduce observed differences. Finally, she had to know how to apply appropriate physics principles to the problem situation in order to produce useful equations. Even more sophisticated domain-specific knowledge was used by the expert solver I observed planning a solution to physics problems. He had to know what kind of features to abstract in constructing a useful simplified problem. He had to know what kind of operations he could apply to solve abstracted problems and how these were to be elaborated when he returned to construct a full solution. (p. 11)

In short, all problem solving strategies are dependent on knowledge and particular procedures so that "one aspect of developing expertise in a discipline would then be the acquisition of less general, but more efficient strategies." (Larkin, 1980, p. 116)

General Problem Solving Research: Any Answers?

Given this situation, is there any way to identify, and eventually assess, general problem solving skills that will have meaning and applicability across areas of human activity? Or would such general skills be so embedded in the particular details of specific fields that there is little point to attempt to isolate them?

Unfortunately, much of the research on problem solving has little bearing on this question for three reasons. First, as Bell (1979) and Scriven (1980) have noted in their examinations of problem-solving research--from very different perspectives--much of the research is based on highly artificial problems, so that its results have little bearing on real-life problem solving.

> Whatever its usefulness elsewhere, most of that literature has to be called irrelevant for our concern with applied problem solving. The level of much of it is indicated by the list of nearly 100 laboratory problem solving tasks in an appendix in Davis (1973). An alphabetical sample will indicate the range: A is for algebra word problems, B is for bent-nail problems, C is for card tricks, E is for

> embedded figure tests, ...M is for matchstick problems,... S is for stick and banana,...W is for water jar problems, and so on. I assure you that there is not much there for "applied problem solving." (Bell, 1979, p. 10)

> The overall design error in much of the work on problem solving to date has been the focus on artificial problems (e.g., rule induction from serial pattern presentation) without any plan for relating this to real-world problems. This is the same pseudo-Galilean approach that kept the learning psychologists in the rat labs for fruitless decades. Galileo had a proof that the results from his experiments in simplified conditions (rolling the ball down the inclined plane) were immediately translatable into answers to the free-fall problem. We have no such proof even at the plausibility level. Studies of the effect of feedback, pretraining, etc. on serial pattern rule induction are <u>at best</u> ways to generate suggestions for research on nonartifical problem solving, and I can see no reason not to approach the problems directly. (Scriven, 1980, p. 136)

However, both Bell and Scriven feel that this research results in occasional insights into the problem solving process but that the insights were <u>provoked</u> by the research rather than <u>demonstrated</u> by the research.

A second difficulty is that the variety of tasks and studies is so great that it is difficult to find any consensus in the literature.

> Research in human problem solving has a well-earned reputation for being the most chaotic of all identifiable categories of human learning. The outstanding quality which leads to this conclusion is the diversity of experimental procedures called 'problem solving' tasks. The tasks found in problem solving literature range from matchstick, bent nail, and jigsaw puzzles through anagram problems, and even include some mental testing devices such as analogy problems and number-series problems. It is almost definitional of laboratory problem-solving experiments that virtually any semi-complex learning task which does not clearly fall into a familiar area of learning can safely be called 'problem solving.' (Davis, 1966, p. 36)

> Much of the research on problem solving proceeds at a deadening low level of empiricism: choose a problem, any problem, and see what variables affect it, in what way.

> Traditional problems seem to be chosen for the same reason that mountains are climbed, because they are there. Except possibly among test constructors, there is almost no prior analysis of what aspects of problem solving are to be isolated for scrutiny, why they are of special interest, or how best to analyze their role in relation to that of other behaviors. There is insufficient concern with validity of tasks, generality of findings, or whether independent variables affect the thought process rather than the context of performance. Admittedly some of these judgments presuppose the existence of a theory, and good general theories of problem solving are not in abundant supply; even a serviceable taxonomy of problems is lacking. (Neimark & Santa, 1975, p. 175)

A third problem comes from the fact that when researchers or specialists in various fields seek to understand and teach the solving of problems, they run into many difficulties in generalizing from one situation to the next. Schoenfeld, for instance, has discussed the considerable difficulties encountered in a course in problem solving for mathematics majors (1979). Although the students learned various heuristics and occasionally used them in attacking various mathematical problems, they had great difficulty in applying them consistently. Schoenfeld concludes "...there is virtually no reliable evidence to indicate that one can substantially enhance students' abilities to solve problems (in any meaningful way) by teaching them heuristics" (p. 65). Larkin (1980), who has attempted to improve problem solving among physics students, has described the difficulties as follows:

> To summarize the situation in the practical classroom then, work in this area suffers from three major difficulties: (1) problem solving is intrinsically very hard to teach, and this is particularly true in the areas of mathematics and science; (2) educational research has traditionally not used methodologies productive in providing information about problem-solving processes; and (3) although some individuals have produced instruction that seems to be effective in helping students to solve problems, very little is known about how this instruction works. What is needed,

> and is not available, is believable research that elucidates the mechanisms of problem solving at a level of detail useful for designing instruction. Current educational work has not really addressed these mechanisms systematically. The best information available consists of the insights of good instructional developers, who often have done considerable informal analysis of the processes required for skillful problem solving in their disciplines." (p. 113)

Thus, there are numerous obstacles in the way of identifying generic competencies in problem solving. However, before we concede defeat, we should note several lines of inquiry that may prove to be fruitful. First, there is some evidence that various heuristics are used by effective problem solvers in many areas of activity when confronted by new types of problems and that these heuristics can be identified. Second, there are converging lines of evidence that a major role is, or can be, played by a "managerial function" that selects strategies and plans attacks on problems. Finally, the study of how problem solvers within specific fields learn to solve the field-specific problems they face suggest several generic skills that cut across fields.

Heuristics or Strategies Used by Effective Problem Solvers

Although we have emphasized the role of knowledge and procedures specific to particular fields, there are a number of strategies that are used by effective problem solvers in various fields. Some of these strategies have also been the subject of experimental psychological research. In general, they are what Newell (1980) has termed "weak methods," mainly used when new or ill-defined problems are encountered. They include such well investigated strategies as means-ends analysis, working backward from the solution, and breaking down the problem into simpler problems that can be solved more easily than the original problem. Means-ends analysis involves examining the current

state or condition of interest, understanding the features of the goal state or solution that is desired, and selecting "operators" or actions that produce certain results to reduce the differences between the current state and the goal state. In general, this involves defining a "problem space" and searching it. This general idea has led to an extensive body of research that has come out of the work of Newell and Simon (1972). This work was originally concerned with the computer simulation of behavior. Due to this approach, a great deal of the work has involved the study of problems that match the kinds of operations that computers perform. These include transforming a block of numbers in a set, like those in the well-known toy which includes movable numbered tiles in a set frame with an empty slot (see Figure 2).

6	4	2
1		7
3	8	5

Figure 2

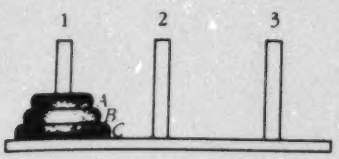
The goal state is a different, but orderly arrangement such as this, in Figure 3.

1	2	3
4	5	6
7	8	

Figure 3

These "move" problems allow tracking of the attempts at solution because each step in the solution is a single change, which can be simulated by the computer. Other problems studied by means-ends analysis include the "Tower of Hanoi"

problem, which involves moving three disks of different sizes from one peg to another. The disks can be moved from one peg to another, but only the top disk on a peg can be moved and it can never be placed on a smaller disk. This problem is more difficult than it looks and becomes much more difficult with the addition of additional disks.



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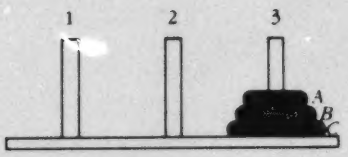

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Analysis of problems like this by Newell and Simon (1972) have led to a computer-simulation of problem solving called the General Problem Solver (GPS). As shown in Figure 4 (taken from Anderson, 1980), the GPS notes the differences between the current state and the desired (goal) state, then breaks the current state into a set of differences and attempts to reduce the differences by applying "operators." Operators are actions that change the current state so that it will be closer to the goal state. If one operator does not produce the desired effect, then a different operator is sought. Obviously, this is a very

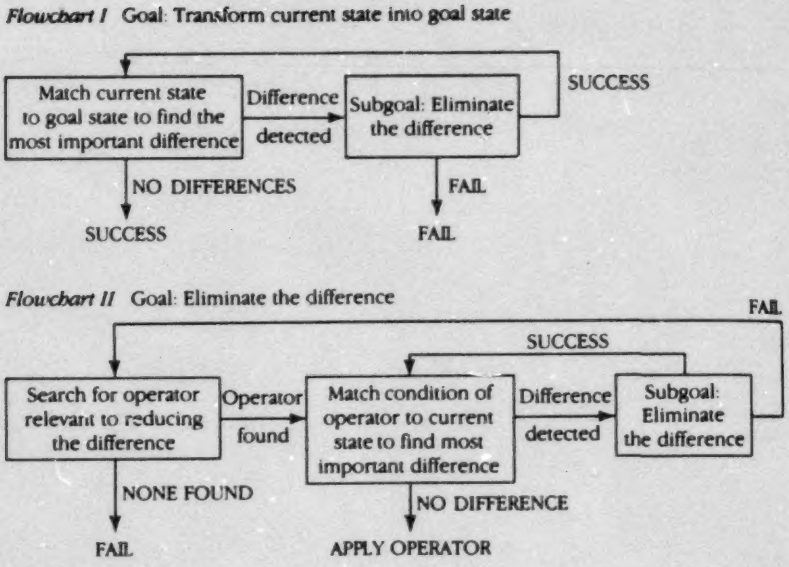

Figure 4. *The application of means-ends analysis by Newell and Simon's General Problem Solver. Flowchart I breaks a problem down into a set of differences and tries to eliminate each. Flowchart II searches for an operator relevant to eliminating a difference.*

abstract representation of human problem solving, and it concentrates on problems that have discrete steps in their solution. However, various researchers have discussed its application to algebra, calculus, and logic problems. Simon and Simon (1978) studied a novice attempting to solve kinematics problems, and found that a means-ends analysis replicated the order in which the problem solver applied various principles of physics. The solver apparently examined the difference between the equations she had and the equation she knew would be required to solve the problem, and attempted to create new equations to reduce the difference. In contrast, an expert solver did not use means-ends analysis, but seemed to think of the problem in terms of a physical representation, to which appropriate principles could be applied. As Larkin (1980) concludes, means-ends analysis is a very general, but inefficient problem solving strategy. "People may apply it to problems for which they have no better method. One aspect of developing expertise in a discipline would then be the acquisition of less general, but more efficient strategies." (p. 16) A related strategy, working backward from the goal, can be especially useful in such activities as finding proofs in mathematics. In general, it is most useful when there are many possible ways to solve a problem but only a few ways actually lead to the solution. Obviously this method applies only when the goal or solution is known and fairly clear.

Another method used in many areas of activity is to simplify or abstract the problem by planning. A classic example is the architect's plan for a building. It is obviously simpler to change conditions on paper rather than to actually build a house, and find it is full of mistakes. The architect's design also abstracts the work and does not take into account the actual things

a builder would have to do, for example, measuring wood for a door frame, adjusting tiles, turning a pipe so it sets properly in its fittings, etc.

Some related and overlapping general strategies have been summarized by Rubenstein (1975) in his book on problem solving. These are listed below as general heuristics for practical use.

1. Avoid getting caught in detail--attempt to see the general pattern or picture. Go over the problem several times until a pattern develops.
2. Withhold premature commitment to a single strategy--consider several before proceeding.
3. Create models--verbalize, make graphs, write down, create abstract or concrete models. A model simplifies the problem.
4. Find new representations of the problem; transform it into another system or context.
5. Question your premises and even reject them, and, if necessary, replace them with others or innovations.
6. Verbalize your situation, ask questions, use different words. This may lead you to recall relevant information from your long-term memory.
7. When the goal is specified, work backward to the beginning.
8. Locate stable substructures that can serve as touchstones in the solution that you can branch out from and return to.
9. Think of and apply analogies and metaphors. This places the problem into a larger structure allowing for solution. (This is the basic approach of such creative problem-solving programs as synectics, Gordon, 1961.)

10. Incubate--leave the problem alone for a while and do something else. Talk to a colleague who may have a different viewpoint that could suggest a solution.

However, as Cyert (1980) points out, the problem with these kinds of heuristics is their unrelatedness. "There is no general theory to guide the student as to the order in which to use these heuristics nor any approach that relates the individual heuristic or subsets of them to particular problems. A raw empirical approach must be used." (p. 7) Furthermore, as Schoenfeld (1979) notes, a problem solver must understand what it means to apply the heuristic. In fact, most of the heuristics are labels attached to closely related families of specific strategies that are appropriate to a given type of problem.

Similar problems apply to various creativity or problem-solving programs. One of the most popular, put forth by Alex Osborn in his book Applied Imagination, 1953, is a "Check list for new ideas." It is reproduced on the following page. Additional words were suggested by Koberg and Bognall (1974) in The Universal Traveler such as:

Multiply	Distort	Fluff-up	Extrude
Divide	Rotate	By-pass	Repel
Eliminate	Flatten	Add	Protect
Subdue	Squeeze	Subtract	Segregate
Invert	Complement	Lighten	Integrate
Separate	Submerge	Repeat	Symbolize
Transpose	Freeze	Thicken	Abstract
Unify	Soften	Stretch	Dissect, etc.

Check List for New Ideas

Put to other uses?
New ways to use as is? Other uses if modified?

Adapt?
What else is like this? What other idea does this suggest? Does past offer a parallel? What could I copy? Whom could I emulate?

Modify?
New twist? Change meaning, color, motion, sound, odor, form, shape? Other changes?

Magnify?
What to add? More time? Greater frequency? Stronger? Higher? Longer? Thicker? Extra value? Plus ingredient? Duplicate? Multiply? Exaggerate?

Minify?
What to subtract? Smaller? Condensed? Miniature? Lower? Shorter? Lighter? Omit? Streamline? Split up? Understate?

Substitute?
Who else instead? What else instead? Other ingredient? Other material? Other process? Other power? Other place? Other approach? Other tone of voice?

Rearrange?
Interchange components? Other pattern? Other layout? Other sequence? Transpose cause and effect? Change pace? Change schedule?

Reverse?
Transpose positive and negative? How about opposites? Turn it backward? Turn it upside down? Reverse roles? Change shoes? Turn tables? Turn other cheek?

Combine?
How about a blend, an alloy, an assortment, an ensemble? Combine units? Combine purposes? Combine appeals? Combine ideas?

Obviously there is no guide to choosing one of these strategies which would be appropriate to any particular problem. Furthermore, these general ideas are so abstract that they can be difficult to apply in concrete terms. They are clearly useful only in situations where the problem solver has no better idea of how to proceed, which brings us to our second area of research.

"Managerial" Functions

Various authors have emphasized the role of "managerial" functions, that choose an approach or method of attacking the problem from the individual's repertoire. Greeno (1980), Sacerdoti (1977), and Miller (1979) have all emphasized the role of planning, with each writer using a very different perspective. In addition, educators in fields involving extensive problem solving have also emphasized the role of planning (e.g., Dadourian, 1949; Larkin, 1980; Polya, 1957; Schoenfeld, 1979). As explained by Greeno (1980):

> Sacerdoti's analysis involved a study of the organization of knowledge about actions at various levels in the form of a procedural network. Each action that is included in the system's knowledge structure is known to have a set of preconditions and a set of consequences, as well as a set of component subactions that are performed in order for the action to be accomplished. This organization permits formation of a plan, beginning with a sequence of global actions and proceeding to more detailed components. (p. 15)

Miller (1979) studied simple programming problems, and emphasized global and local processing in planning. The problems are decomposed into manageable subgoals, and a sequence of planning-debugging-planning-debugging was identified. Greeno (1980) has considered planning to be one of the broad dimensions of problem-solving skill, although he notes that the particular planning strategy may be specific to a domain of problems:

I think that we have to conclude that planning, like problem solving, is not the kind of process that either does or does not occur in a situation. Instead, it probably occurs in different ways, depending on the knowledge that the problem solver has about the domain. If a person has a rich and well organized structure of knowledge about actions in the domain, planning seems to occur as a routine form of problem solving similar to that found in ordinary problems of arrangement such as cryptarithmetic or anagrams. Planning by a novice may be more generative, but it seems likely that if a person wishes to become skilled in the kinds of problem solving that anticipate difficulties that can arise in a domain, the acquisition of a well-organized procedural network for the domain may be more useful than the acquisition of general planning procedures.

Dadourian (1949), in his pamphlet designed for mathematics students "How to Study, How to Solve," says:

> The formulation of a plan of action is the most important part of the solution of difficult problems. When the right strategy is thought out the problem is virtually solved and the actual execution of the strategy becomes a matter of careful and orderly work. The student should solve some problems completely by carrying out the necessary detailed analysis, while others he should solve only mentally by formulating a plan of action. In this way he can obtain a greater mastery of the subject without too much expenditure of time. (p. 21)

Finally, Schoenfeld (1979) who calls planning DESIGN, describes its importance in solving mathematical problems.

> At the simplest level (which obtains for straightforward or routine problem solving) DESIGN consists "merely" of the intelligent ordering and structuring of an argument. The problem solver should have an overview of the solution process; he should be able to say, at any particular point in the process, what he (or she) is doing, why he is doing it, and how that action relates to the rest of the solution. He should proceed through (all but the most routine) solutions hierarchically, taking care to avoid being immersed in intricate calculations pertaining to one part of a solution if global aspects of another phase of problem solution remain unresolved. (We have all suffered the discomfort of solving a difficult equation, only to discover that it didn't have to be solved in the first place!)

> For more complex problems, however, DESIGN takes on more global and significant dimensions. It is different from the other phases of problem solving, in a sense pervading them all. Design is a "master control," monitoring the whole of the problem-solving process, and (as best it can with the information it has) allocating problem-solving resources efficiently. It keeps track of alternatives, so that if the chosen approach to a problem proves more difficult than expected, other approaches to the problem can be considered and (in the light of this difficulty) the most likely to succeed chosen. If there is difficulty in making a straight-forward plan, DESIGN sends the problem solver into EXPLORATION. Problems resolved without much difficulty are returned to DESIGN and the elaboration of the problem-solving plan continues. However, if EXPLORATION provides new insights into the problem or the solution process, the "master control" may decide that it is most appropriate to return the problem, with the new information, to ANALYSIS. (p. 50)

At a more theoretical level, Sternberg (1981) has postulated an "executive function" that calls on the appropriate components of intelligence and applies them in appropriate ways when the individual is faced with a task.

All of these ideas suggest the importance of formulating a plan and the difficulties of proceeding without one. Many studies of problem-solving, from Bloom and Brode (1950) on, show that poor problem solvers frequently flounder about, trying various possibilities almost at random. Effective problem solvers frequently try to think of a sequence of coordinated activities that may lead to a solution. Although planning does not guarantee a solution, it almost certainly increases the probability of finding a solution. By consciously spelling out one's assumptions, outlining possible routes to the solution, and by checking to see whether an idea fits the requirements of the problem situation, one can test hypotheses and revise them, and thereby systematically proceed toward a solution. As with all the other problem solving abilities we have considered, it is obviously important to have knowledge of the field in which the problem exists. However, it seems possible that training in general planning skills would lead to higher hit rates in solving problems.

Problem Solving Within Disciplines

The third line of evidence deals with analyses of how problems are solved within various disciplines. Several of the authors mentioned earlier have described the problem-solving process in their disciplines. Furthermore, although they may not be called "problem-solving" the procedures of many fields are designed to deal with real life problems. For example, engineering curricula deal with many kinds of problems. (Gordon, 1961, and Rubenstein, 1980, describe general strategies in engineering and other technical fields.) Larkin (1980) has described problem-solving in physics. Medical problem solving has a literature of its' own (see Elstein, Schulman, & Sprafka, 1978 for a review). Some of the most pertinent research has been conducted by Christensen-Syalanski (e.g., Christensen-Syalanski & Bushyhead, 1981.) The field that has devoted the greatest attention to problem-solving strategies is mathematics (e.g., see the volumes by Hill [1979]; Lesh, Mierkiwicz, & Kantowski [1979] and Krulisk & Reys [1980]). Stemming from the original work by Polya (1957), there is a vast literature on problem-solving strategies, characteristics of problems and teaching for problem-solving. Although the evidence on the extent to which problem-solving can be taught encourages humility, this research has identified various skills that are important, and which appear to have sufficient generality as to apply across fields. For example, Suydam (1980), drawing on writers who brought "... their individual perspectives to bear, ranging from pragmatism to the sophistication of artificial intelligence models" proposes the following elements in problem-solving:

1. Understanding the problem--an awareness of the problem situation that stimulates the person to generate a statement of the problem in writing, orally, or merely in thought.

2. Planning how to solve the problem:
   a) Break down the components; enumerate data; isolate the unknown.
   b) Recall information from memory; associate salient features with promising solution procedures.
   c) Formulate hypotheses or a general idea of how to proceed.
3. Solving the problem:
   a) Transform the problem statement into a mathematical form, or construct representations of the problem situation.
   b) Analyze the statement into subproblems for which the solution is more immediate.
   c) Find a provisional solution.
4. Reviewing the problem and the solution:
   a) Check the solution against the problem.
   b) Verify whether the solution is correct; if not, reject the hypotheses, the method of solution, or the provisional solution.
   c) Ascertain an alternative method of solution. (p.40)

This, and research in many other fields, suggest that various generic skills or abilities are important across fields. They are outlined below along with some discussion of related abilities.

1. The ability to identify a problem and to state its components. Obviously this depends on familiarity with a domain of problems and experience with similar problems. Such familiarity allows the solver to "chunk" the elements of the problem--i.e., to see meaningful patterns which suggest solutions. The classic models of this behavior are chess

masters. Their powers of logic and the number of moves they plan ahead are not greater than those of amateurs. However, they are immensely superior in their ability to recognize meaningful patterns. Simon and Gilmartin (1973) estimate that masters can recognize on the order of 50,000 chess patterns. Ironically, one of the outcomes of acquiring skill is to reduce the supposed degree of creativity involved. "To become an expert in any problem-solving field requires years of study. The effect of this study is to transform solution by creative problem solving into solution by the simple retrieval of stored answers. One becomes an expert by making routine many aspects of a problem that require creative problem solving for novices. Thus, one's behavior is less error prone and more attention can be focused on those aspects of the problem that cannot be routinized." (Anderson, 1980, p. 292).

2. The ability to formulate a plan to attack the problem. Again, this ability is based on familiarity with a problem domain and experience with similar problems. The novice in an area will probably have to use such general strategies as means-end analysis, breaking the problem into subproblems, or setting subgoals, simplifying, and working backwards. The expert will have a number of more domain-specific but more efficient strategies. However, the more efficient problem solver, whether novice or expert, should have a repertoire of possibly applicable strategies and be able to formulate a plan to use them.

3. Having requisite knowledge and the ability to recall it and associate relevant features with the current problem. This is quite obvious. One cannot solve problems in chemical engineering without some knowledge of

chemistry. However, the capacity to search long-term memory for relevant information probably varies, even among people who are quite knowledgeable in a problem domain. Likewise the ability to associate the knowledge with the demands of the current situation probably varies. In fact, Mednick (1962) considers the creative thinking process as the forming of associative elements into new and useful combinations, and Maltzman (1960) believes that originality can be trained by increasing the number and remoteness of responses. Related abilities are the ability to see what additional information is necessary for a solution and the ability to identify and disregard information that is extraneous to the central problem.

4. The ability to formulate hypotheses about the problem and/or a plan to eliminate various possible solutions. Within certain domains, especially scientific disciplines, this ability is critical. Wason (1968) has suggested that the inability to use hypotheses, especially the difficulty in dealing with negative instances, is a major stumbling block for many problem-solvers. Obviously, the scientific method and experimental design are the most formal expressions of this capacity. (However, it is often the case that scientific methodologists who are experts in the techniques of testing hypotheses and statistical methods do not produce hypotheses of their own to examine. This may be a critical test of the idea that training in general problem solving techniques does not necessarily lead to the ability to solve real life problems without knowledge of the specific field involved.) It is especially important to understand the structure and central concepts of a field, not just its techniques. That is, to be able to generate

hypotheses, it is essential to understand why certain effects are probable. Thus, a physician who understands the physiological mechanisms underlying a drug's effects can prescribe drugs more effectively. Less formal versions of hypothesis testing are "trouble shooting," diagnosis, etc. Once again, knowledge, and especially understanding of a domain is critical to effective generation of multiple hypotheses and the use of hypothesis testing. However, as is clear from studies of scientists, some people make much more effective use of their knowledge to create and test hypotheses than others in the same field. (The interviews with eminent scientists in the book The Way of the Scientist, selected by the editors of the journal International Science and Technology, provide a great deal of anecdotal and descriptive evidence about the various influences on scientists' capacity to generate and test hypotheses. The interviews also provide many stimulating insights into the processes of high level problem solving and creativity, as the scientists describe their own thinking and actions as they attempted to solve problems, formulate hypotheses and find means for testing them. The interviewees include such diverse scientists as Leo Sziland, Albert Szent-Gyorgyi, Glenn Seaborg, and C. P. Snow.)

5. The ability to construct a representation of the problem situation. This includes the capacity to see the problem in abstract terms. Frequently, when a problem is seen in terms of basic relationships, with lessened attention to details, the relationships make it easier to see the solution. A related capacity is that of developing models of the problem situation. In some cases the models may be actual physical representations of the situations, graphical depictions,

or logical maps. Models allow relatively easy manipulation of the elements of a problem and clearer understanding of the structure of relationships among them.

6. The ability to see the sequence of steps of activities and subproblems needed to reach the solution; fitting the subproblems into a general pattern. Although this capacity is related to planning, described above, this is more related to sequencing activities; i.e., seeing what needs to be done first, and how a set of apparently disconnected activities can be organized into a concerted effort at a solution.

7. The ability to formulate a provisional solution. In many fields, it is necessary to go beyond the formulation of hypotheses, described above, to formulating a possible solution. That is, after one hypothesizes about the possible causes and influences on the problem situation, it is necessary to formulate a solution to the problem, based on an understanding of the causes and factors influencing the situation. Obviously creativity plays a major role in this ability, as do familiarity with similar or analogous problems. Various investigations into the use of analogies and metaphors to solve problems have been conducted recently (Weisberg, 1980). Although this research has obtained various results, it appears that most people have difficulty in using analogies and metaphors as a way of suggesting solutions to problems that are not highly similar to the problem presented in the analogy.

8. The ability to check the attempted solution against the problem's requirements and to see the correctness of the solution. Again, as noted in the classic work of Bloom and Brode (1950) many people have difficulty even seeing why a solution is a solution. Elements of

this ability include the capacity to see similarities and differences between the attempted solution and the desired state, being able to state the requirements of the problem and assess the extent to which the proposed solution meets these requirements. Again, knowledge of a domain allows one to understand why a problem was solved. This understanding in turn may allow the construction of more efficient or elegant solutions.

9. There are also various personal traits that are frequently suggested as characterizing the good problem solver. These include a combination of perserverance and flexibility. Some poor problem solvers tend to give up on problems if they cannot reach a solution almost immediately. Others cannot consider more than one approach to the problem and will not abandon it even if it is clearly inappropriate. In contrast, good problem solvers stay with problems, and entertain a variety of possible solutions until they reach a solution or it is clear that there is no way to solve the problem. Another trait is open-mindedness to new ideas and approaches. This is related to another characteristic, skepticism or willingness to doubt current approaches to problems.

Application of these Competencies

The answer to the central question of this review appears to be that there are problem solving strategies and skills that are used commonly across fields. However, it is clear that the implementation of these strategies and skills depends on knowledge of the domain and that the procedures needed to implement them are largely determined by the field. However, it appears probable that exercises that assess problem-solving skills within fields could be constructed. Several considerations bear on such assessments.

First, as noted earlier in the discussion of heuristics, different kinds of problems will require different competencies to different degrees. Problems in different domains of activity--and different problems within those domains--will require these competencies to different degrees. Further, these competencies will probably be called on to different degrees as a problem-solver becomes more expert in the solution of problems within his or her field. The novice medical student may find the process of generating a plan for diagnosis most critical in diagnosing a patient's disorder. In contrast, the expert physician may find the generation of hypotheses most important (Elskin, Schulman & Sprafka, 1978). Furthermore, these competencies are quite abstract. Their implementation is highly content specific.

Applying these problem solving tactics is difficult and requires extensive experience in a field. In addition, it is unclear whether these skills generalize from one type of problem to another type within a field. And, as noted many times in this review, it is even unclear whether they can be effectively taught to novices in the field. However, it seems probable that some people are more effective in using these tactics than others, whether due to training or intrinsic talent. We have all known people with a "knack" for fixing cars, solving chemistry problems or dealing with ticklish interpersonal situations. It is possible that such people could be identified in specific fields by presenting them with excercises that are designed to assess their skill in using the tactics we have outlined. For example, exercises in business problems could be developed for applicants to schools of management. Some early work along these lines was done by Frederiksen, Saunders, and Wand (1957) in their "in basket test," which simulates various

problems an executive might face by presenting the person with memoranda, letters and documents dealing with typical business problems.

Another example is the work of Frederiksen and Ward (1978) in assessing prospective scientists' capacity for dealing with ill-structured problems.

One area of promise that was not covered extensively in this review is that of computer simulations of problem-solving behavior. That is, the computer is programmed to solve problems in the same ways as an expert human problem solver. These can range from the simulation of checker and chess games, to the diagnostic situations faced in emergency rooms. Some programs aid the problem solver by suggesting the logical or procedural steps the effective human problem solver usually would use in solving the type of problem presented. A great deal of recent effort has been done in the area of programming computers to ask questions and process the answers. For example, a program was written to help scientists analyze the rocks brought back from various moon expiditions, LSNLIS (Lehnert, 1978). Another program for more prosaic functions is GUD (Bobrow et al., 1977), which is an interactive dialogue program designed to assume the role of a travel agent in a conversation with a client. These and other programs are discussed in Lehnert (1978). The general considerations involved in using computer and information systems to help solve problems and do a variety of other things are discussed in Schneiderman (1980).

Computers offer the potential of simulating problem situations in a fairly short time. For example, a computer could be programmed to provide the sorts of exercises used by Alderman, et al. (1980), yielding scores according to both the accuracy and efficiency of the solution. However, the computer could also hold relevant data in its memory, provide hints, and answer questions, if

they were requested by the person being assessed. The pattern of inquiry could be scored for the general effectiveness of the strategy used, as well as such variables as the relevance of the data requested, appropriateness of questions asked of the data, etc.

Any assessment would need to be field-specific in content--e.g., presenting candidates to graduate management schools with common business problems, or prospective law students with situations faced by lawyers in their daily work (see Alderman, Evans and Wilder (1980) for an example of a paper and pencil method for assessing skills used in the profession of law). This will probably be simpler in some fields than others. For example, the GRE Advanced tests in various disciplines are probably fairly close to the actual work in the fields assessed. However, in other fields such techniques as video-taped situations or physical examples may be needed.

These exercises could be scored for the skills we have identified in the previous pages--e.g., the ability to state the problem, the ability to structure an overall plan, the ability to formulate competing hypotheses, etc. Obviously, a great deal of care would be needed to construct such exercises so that they would actually resemble typical problems in each field, would have face validity, and would be generalizable to the solution of many problems in the field. The exercises and the scores would need to have sufficient reliability across candidates to be useful. They would also need to have validity--a requirement that presents a variety of difficulties. One simple way to demonstrate validity would be to show that professionals in the field score better than amateurs. This strategy was used by Alderman, Evans and Wilder (1980) who showed that there was a logical progression of scores on their simulations of legal problems: undergraduates, law school students, lawyers, and law school professors. A more

difficult, but more convincing demonstration of validity would be a study showing that such measures of problem-solving skill predict subsequent professional performance. In any case, the measures would almost certainly have to be developed separately for different fields, such as business, science, law, etc. The basic underlying problem-solving skills might be the same, but the content would need to be fitted to the field. All of this would require a good deal of research, such as that reported by Elstein, Schulman, and Sprafka (1978) in medical problem solving, but the end product might be worth the price.

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